# Mid-Term Examination

## Algebra IV

## B.Math.

1st March, 2010

**Instructor: Shreedhar Inamdar**

**Instructions.** All questions carry equal marks. $\mathbb{Q}$ denotes the field of rational numbers.

**1.** Define the degree of a field extension and prove that it is 'multiplicative'.

**2.** Let $F$ be a finite extension of $\mathbb{Q}$. Prove that the algebraic closure of $F$ and $\mathbb{Q}$ are isomorphic to each other.

**3.** Let $K$ be a finite separable extension of $F$. Let $G$ denote the group of automorphisms of $K$ over $F$. Prove that $\{x \in K \mid g(x) = x, \forall g \in G\}$ equals $F$.

**4.** Describe the degrees of the splitting fields over $\mathbb{Q}$ of the polynomials $X^3 + 2X + 1$ and $X^5 - 3$. Justify your answer.

**5.** Let $f$ be a degree 6 irreducible polynomial over a field $F$. Let $K$ be a degree 2 extension of $F$. Prove that either $f$ is irreducible over $K$ or is a product of two irreducible cubic polynomials over $K$.

**6.** Define separable degree of a field extension. Prove that the separable degree of an finite extension is bounded by the degree of that extension.